# Rapid Method for Determining Concentric Cylinder Radiation View Factors

Samuel N. Rea*
*Texas Instruments Incorporated, Dallas, Texas*

## Nomenclature

$F_{di-j}$ = view factor from elemental area $i$ to surface $j$
$F_{i-j}$ = view factor from surface $i$ to surface $j$
$F_{(Y-a)}$ = shorthand notation for cylinder-disc view factor, Fig. 3, with $\ell/r=(Y-a)/r$, etc.
$r$ = inner cylinder radius
$R$ = outer cylinder radius

## Introduction

CONCENTRIC cylinders are among the more common configurations encountered in radiation heat transfer analyses, being descriptive of assemblies ranging from infrared telescopes to tube furnaces. In performing thermal analyses on such structures with finite-element computer programs, it is often necessary to break the shell and tube into various concentric ring elements and compute radiation view factors between these elements. Figure 1 depicts a collection of concentric cylindrical geometries. In general, the tube-shell view factors of Fig. 1 can be determined using available computer programs which numerically integrate over the areas involved. However, these programs are cumbersome, thereby limiting their casual usage.

A recent paper by Reid and Tennant[1] numerically analyzed configuration IV in Fig. 1 for the special case where $L=Y$. The more general case, along with the other configurations in Fig. 1, is not in the open literature, although the equal-length case is given in Ref. 2. The novelty of the method presented in this Note lies in the utilization of the cylinder-disk view factor $F_{1-2}$ of Figs. 2 and 3. By combining various cylinder-disk view factors, for any of the structures in Fig. 1 $F_{1-3}$ can be determined in closed form where it is the diffuse view factor from the outer curved surface of the inner tube to the outer cylinder. The view factors from the ends of the inner cylinder to the outer cylinder in configurations II, III, and IV are not considered since they can be determined using disk-disk relations[2] as can the shell-shell view factors.[1] Of course, $F_{3-1}$ can be readily computed from $F_{1-3}$ using reciprocity.

## Analysis

A literature survey indicates that the cylinder-disk view factor is not explicitly available so it is derived first and then applied to the Fig. 1 structures. By definition,[3] with the geometry of Fig. 2:

$$F_{1-2}=\frac{1}{\ell}\int_0^{\ell} F_{d1-2}\,\mathrm{d}x \qquad (1)$$

where $F_{d1-2}$ can be derived from Ref. 4:

$$2\pi F_{d1-2}=\cos^{-1}\frac{x^2-R^2+r^2}{x^2+R^2-r^2} - \left(\frac{x}{r}\right)\left[\frac{x^2+R^2+r^2}{[(x^2+R^2+r^2)^2-4R^2r^2]^{1/2}} \times \cos^{-1}\frac{r(x^2-R^2+r^2)}{R(x^2+R^2-r^2)} - \cos^{-1}\left(\frac{r}{R}\right)\right]$$

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*Engineering Supervisor.

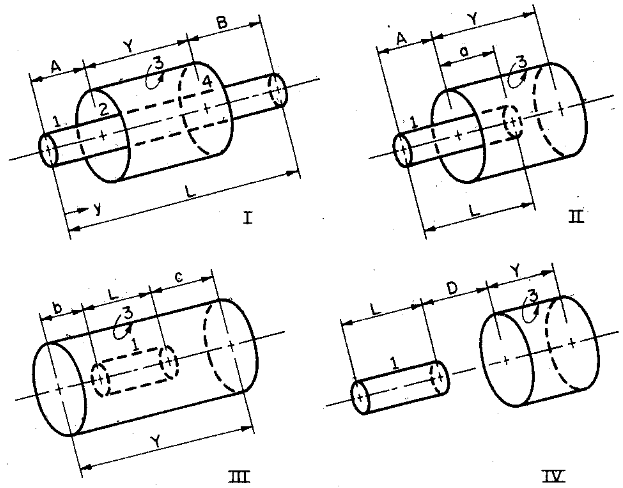

Fig. 1 Concentric cylinder configurations.

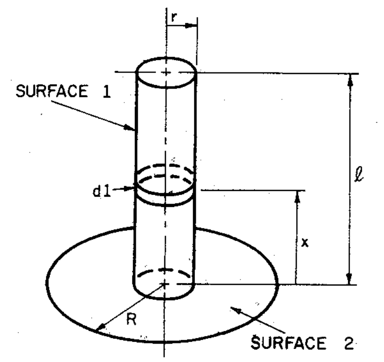

Fig. 2 Cylinder-disk view factor geometry.

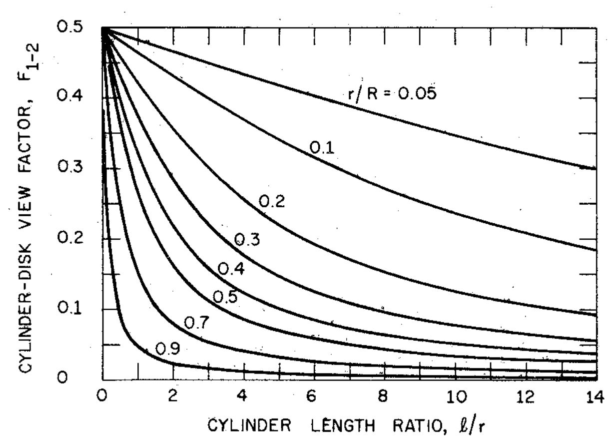

Fig. 3 Cylinder-disk view factors.

The integration in Eq. (1) is given by

$$2\pi F_{1-2}=\cos^{-1}\frac{\ell^2-R^2+r^2}{\ell^2+R^2-r^2} - \left(\frac{r}{2\ell}\right)\left[\left[\frac{(\ell^2+R^2+r^2)^2}{r^4}-4\left(\frac{R}{r}\right)^2\right]^{1/2} \times\cos^{-1}\frac{r(\ell^2-R^2+r^2)}{R(\ell^2+R^2-r^2)} + \left[\frac{\ell^2-R^2+r^2}{r^2}\right]\sin^{-1}\left(\frac{r}{R}\right) - (\pi/2)\left[\frac{\ell^2+R^2-r^2}{r^2}\right]\right] \qquad (2)$$

The cylinder-disk view factor $F_{1-2}$ given by Eq. (2) is presented in parametric form in Fig. 3.

The view factors $F_{1-3}$ for the various structures in Fig. 1 are presented in the following. The derivation for configuration I is shown in some detail to illustrate the procedure. In all cases $L$ is the length of the inner cylinder and $Y$ is the length of the outer cylinder. Other dimensions are shown in Fig. 1.

**Configuration I**

Again, by definition[3] and referring to Fig. 1

$$L F_{1-3} = \int_0^L F_{d1-3} \mathrm{d}y \quad (3)$$

By introducing appropriate view factor algebra, Eq. (3) can be written

$$L F_{1-3} = \int_0^A (F_{d1-2} - F_{d1-4}) \mathrm{d}y$$

$$+ \int_A^{A+Y} (1 - F_{d1-2} - F_{d1-4}) \mathrm{d}y + \int_{A+Y}^{L} (F_{d1-4} - F_{d1-2}) \mathrm{d}y \quad (4)$$

In Fig. 1 and Eq. (4), surfaces 2 and 4 are imaginary annular disks covering the ends of the outer cylinder. A term-by-term comparison of the integrations indicated in Eq. (4) with the cylinder-disk definition, Eq. (1), shows that the solution of Eq. (4) can be written immediately in terms of the cylinder-disk view factors given in Fig. 3 or by Eq. (2). After som algebraic manipulation

$$LF_{1-3} = Y + AF_A + BF_B$$

$$-(A+Y)F_{(A+Y)} - (B+Y)F_{(B+Y)} \quad (5)$$

A view factor shorthand notation is used in Eq. (5) and below where, for instance, $F_A$ is the cylinder-disk view factor $F_{1-2}$ with $\ell/r = A/r$.

**Configuration II**

$$LF_{1-3} = AF_A + a(1 - F_a)$$

$$+ (Y-a)F_{(Y-a)} - (A+Y)F_{(A+Y)} \quad (6)$$

**Configuration III**

$$LF_{1-3} = L + bF_b + cF_c - (L+b)F_{(L+b)} - (L+c)F_{(L+c)} \quad (7)$$

**Configuration IV**

$$LF_{1-3} = (L+D)F_{(L+D)} + (Y+D)F_{(Y+D)}$$

$$-DF_D - (L+D+Y)F_{(L+D+Y)} \quad (8)$$

## Discussion

A quick check on the validity of the previous equations can be obtained by examining some limiting cases. Configurations I and III reduce to the identical result when $A = B = b = c = 0$ in which case both Eqs. (5) and (7) give $F_{1-3} = 1 - 2F_L$, the equal-length concentric cylinder case. Equations (5) and (6) agree when $B = 0$ and $a = Y$ as do Eqs. (6) and (8) with $a = D = 0$. Also, Eq. (8) agrees with results computed in Ref. 1.

Any of the view factors given by Eqs. (5-8) can quickly be determined by reading the cylinder-disk factors from Fig. 3. Comparisons with computer programed results for several of the configurations have indicated that 2-significant-figure accuracy or better can be obtained consistently using Fig. 3 plotted on graph paper. The technique presented here is also applicable to a concentric inner cylinder and outer truncated cone with slight modification.

## References

[1] Reid, R. L. and Tennant, J. S., "Annular Ring View Factors," *AIAA Journal*, Vol. 11, Oct. 1973, pp. 1446-1448.

[2] Siegel, R. and Howell, J. R., *Thermal Radiation Heat Transfer*, McGraw-Hill, New York, 1972, Appendix C.

[3] Siegel, R. and Howell, J. R., *Thermal Radiation Heat Transfer*, McGraw-Hill, New York, 1972, p. 187.

[4] Leuenberger, H. and Person, R. A., "Compilation of Radiation Shape Factors for Cylindrical Assemblies," ASME Paper 56-A-144, 1956.

# Effect of Thermal Gradient on Frequencies of Tapered Rectangular Plates

C. Kameswara Rao*

and

B. Satyanarayana†

*Andhra University, Waltair, Andhra Pradesh, India*

## Nomenclature

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|---|---|
| $a$ | = length of the plate |
| $b$ | = width of the plate |
| $C_i$ | = constants of linear combination |
| $\bar{D}(x)$ | = flexural rigidity variation |
| $E$ | = modulus of elasticity |
| $p$ | = circular frequency of vibration |
| $r$ | = $\pi a/b$, a parameter |
| $T$ | = temperature excess above a given reference |
| $\bar{W}(\bar{x},\bar{y},t), W(x,y)$ | = lateral deflections of plate |
| $\bar{x},\bar{y}$ | = coordinates in the plane of the plate |
| $x$ | = nondimensional coordinate |
| $\alpha$ | = $\gamma T_0$, a parameter |
| $\lambda$ | = $\rho a^4 h_0 p^2/gD$, eigenvalue relating to frequency |
| $\nu$ | = Poisson's ratio |
| $\rho$ | = mass density of the material of the plate |
| $\emptyset(X)$ | = function related to plate deflection |

## Introduction

CONSIDERABLE work has been done on the vibrations of uniform and tapered rectangular isotropic plates.[1-7] It is well known,[8] that in the presence of constant thermal gradients the elastic coefficients of homogeneous materials become functions of space variables. Recently, Fauconneau and Marangoni[9] studied the effect of the nonhomogeneity caused by a thermal gradient on the natural frequencies of simply-supported plates of uniform thickness. Upper and lower bounds are computed using the Rayleigh-Ritz method and the Bazley-Fox second projection method,[10] respectively. The present investigation is to study the effect of a constant thermal gradient on the frequencies of tapered rectangular isotropic plate which is simply-supported on one pair of edges and with combinations of clamped and simply supported conditions on the other pair of edges. Design formulas for the fundamental frequency parameter are derived by using Galerkin's method. Results for uniform simply supported plates compared well with those of Ref. 9.

## Analysis

It is assumed that the tapered plate is of isotropic material subjected to a steady one-dimensional temperature

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*Lecturer, Department of Mechanical Engineering.
†Postgraduate Student, Department of Mechanical Engineering.